## MTH 111, Math for Architects, Exam I, Summer 2013

Ayman Badawi

QUESTION 1. Given $12 x=y^{2}-4 y-20$. Find the vertex, the focus, and the directrix, then sketch

QUESTION 2. Let $v=2 i+4 j-3 k$ and $w=5 i-7 j+c k$. Given $v$ is perpendicular to $w$. Find $c$.

QUESTION 3. a) Find an equation of the plane that passes through $(2,4,0),(2,6,2),(-1,3,3)$.
b) Does the vector $w=3 i-6 j+2 k$ lie in the plane in (a)?
c) Find the distance between the plane $2 x+2 y-z=8$ and the point $(1,1,-1)$

QUESTION 4. Find an equation of the ellipse with foci : $(4,2),(-2,2)$ such that the point $(6,2)$ lies on the curve of the ellipse. Then find the ellipse-constant, and sketch.

QUESTION 6. a) given $x^{2}+4 x-4 y^{2}-8 y-4=0$ is an equation of a hyperbola. Find the foci, the vertices, the center, and the hyperbola-constant, then sketch.
b) Find a parametric equations of the line that passes through $(2,4,1)$ and $(4,5,6)$.
c) (Extra Credit 3points) : Name the Arab Idol 2013.

## Faculty information

Ayman Badawi, Department of Mathematics \& Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

## MTH 111, Math for Architects, Exam II, Summer 2013

Ayman Badawi

QUESTION 1. Given $L_{1}: x=2 t+3, y=3 t-1, z=-4 t+9, L_{2}: x=3 s-2, y=s+2, z=2 s-5$ intersect in a point $Q$. Find $Q$.

QUESTION 2. Give $P_{1}:-3 x+2 y+z=10, P_{2}: 5 x+2 y+11 z=12$ are two planes.
a) Show that $P_{1}$ is perpendicular to $P_{2}$.
b) $P_{1}$ intersects $P_{2}$ in a line $L$. Find a parametric equations of $L$.

QUESTION 3. a)Find two numbers $a, b$ such that $a+4 b=34$ and $a b$ is maximum.
b)We want to construct a rectangle with maximum area above the line $\mathrm{y}=2$ and inside the curve $y=10-x^{2}$ so that two vertices on the line $y=2$ and the other two vertices on the curve $y=10-x^{2}$. What should be the length and width of the rectangle?

QUESTION 4. Find $y^{\prime}$ and do not simplify:
a) $y=5 \sqrt{3 x^{2}+6 x+1}$
b) $y=4 x^{3}+\frac{6}{x^{2}}+10 x+5$
c) $y=(3 x+2)\left(x^{3}+6 x+3\right)$

QUESTION 5. Let $y=12 \sqrt{x+1}+2 x-4$.
a) Find an equation of the tangent line to the curve at the point $(3,26)$.
b) Approximate the $y$ value when $x=2.6$

QUESTION 6. Let $k(x)=2 x^{3}+3 x^{2}-12 x+20$ defined on $[-3,2]$. Find the absolute maximum (absolute minimum) value of $k(x)$.

QUESTION 7. a) $\lim _{x \rightarrow 2} \frac{\sqrt{x+2}-x}{x-2}$
b) $\lim _{x \rightarrow-3^{-}} \frac{x+4}{x+3}$
с) $\lim _{x \rightarrow-1} \frac{x^{2}-1}{x^{2}+7 x+6}$

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Ayman Badawi, Department of Mathematics \& Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

## MTH 111, Math for Architects, Exam III, Summer 2013

Ayman Badawi

QUESTION 1. i) Let $w=-2 i+2 j+k$. Find a vector $v$ such that $\|v\|=1.4$ and $v \| w$.
ii) Given the line $L: x=t+1, y=-2 t+1, z=t+1$. Does $L$ intersect the plane $P: x+5 y+z=23$ in exactly one point? if yes, then find the intersection-point.
iii) Given the line $L: x=3 t+5, y=2 t+4, z=2 t+1$. Does $L$ lie in the plane $P: 2 x-y-2 z=4$ ? explain

QUESTION 2. Let $f(x)=2 x e^{(3 x-2)}-2 e^{(3 x-2)}+\frac{8}{3}$.
i) For what values of $x$ does $f(x)$ increase?
ii) For what values of $x$ does $f(x)$ decrease?
iii) Find all local minimum (maximum) values of $f(x)$.

QUESTION 3. Given two points $A=(8,2), B=(4,5)$. Find a point on the horizontal line $y=1$, say $C$, such that $\|A C\|+\|C B\|$ is minimum.

QUESTION 4. Given $4 e^{y-2}+x y+x^{2}+4 y+2 x+2 y^{2}-25=0$. . Find an equation of the tangent line to the curve at the point $(1,2)$. Then approximate the $y$-value when $x=0.8$

QUESTION 5. A particle moves on the curve $x^{2}+2 y^{2}+4 e^{2 y-2}+8 x-26=0$, such that the rate of change of $x$ is decreasing by $0.3 \mathrm{~cm} / \mathrm{sec}$. Find the rate of change of $y$ at the point $(2,1)$.

QUESTION 6. Let $f(x)=-2 x^{3}+12 x^{2}+2$. Sketch $f(x)$ but first determine when $f(x)$ is increasing (decreasing), local min.(max) values of $f(x)$, when the curve is concave up (down), and inflection point.

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Ayman Badawi, Department of Mathematics \& Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

# MTH 111, Math for the Architects, Exam One 

Ayman Badawi

QUESTION 1. ( 12 points) Find the focus, the vertex, and the directrix for the parabola $8 y=4 x^{2}+8 x+20$. Let $F$ be the focus of the given parabola. Given $A=(1,4)$ lies on the parabola. Find $|A F|$. Drew a rough graph of the given parabola.

QUESTION 2. ( 12 points) Given $(3,9)$ and $(3,-7)$ are the foci of a hyperbola and $K=6$ is its constant. Write down the standard form equation of the hyperbola. Sketch a rough graph of the hyperbola.

QUESTION 3. ( 12 points) Find the foci, the center, and the constant $K$ for the ellipse $9 x^{2}+5 y^{2}+20 y-25=0$. Sketch a rough graph of the ellipse.

QUESTION 4. ( 6 points) Does the line $y=x+2$ intersect the hyperbola $y^{2}-(x-1)^{2}=3$ ? If yes, find the intersection points.

QUESTION 5. a) ( $\mathbf{1 0}$ points) Find the equation of the line that is perpendicular to the line $3 y+4 x=2$ and it passes through the point $(4,1)$.
b)( 5 points) Given $L: 2 x+3 y=13$ and $A=(4,6)$ is a point not on the line $L$. Find the distance between $A$ and $L$.

QUESTION 6. (i) ( 5 points) $\operatorname{Lim}_{x \rightarrow-1} \frac{\overline{\sqrt{x+1}}-3}{x^{2}-1}$
(ii) $\left(5\right.$ points) $\operatorname{Lim}_{x \rightarrow-2^{+}} \frac{x+3}{x^{2}-4}$

## Faculty information

1. (5pts) Find the equation of the line that passes through the point $(1,2)$ and is perpendicular to the line $6 x-2 y=3$.
2. (5pts) Find the value of $k$ such that the lines

$$
\begin{aligned}
\mathbf{r}_{1}(t) & =\langle 4,-9,1\rangle+t\langle 2, k, 1\rangle \\
\mathbf{r}_{2}(t) & =\langle 4,-9,1\rangle+t\langle 4,6,2\rangle
\end{aligned}
$$

are parallel
3. (5pts) Find the parametric equation of the line passing through the points $(1,0,4)$ and $(1,1,2)$.
4. (5pts) Find the equation of the plane containing the points $(0,0,1),(2,0,1)$ and $(1,1,2)$
5. (5pts) Find the equation of the line which passes through the point $(3,5,7)$ and is perpendicular to the plane $2 x+4 y+6 z=8$
6. (10pts) For each of the following equations state the shape of the curve it determines (eg, line, circle, etc). You do NOT have to provide any reasoning.
(a) $x+y=4 y$
(b) $4 x+y^{2}=7$
(c) $y^{2}=4+x^{2}$
(d) Polar equation $r=\theta$.
(e) Polar equation $r-2+\cos (\theta)$.
7. (5pts) Find the equation of the hyperbola so that: it is centered at the origin, one of the focal points is $(0,5)$, and the distance between the axis intercepts is 6 .
8. (5pts) Carefully sketch the polar curve $r=\cos (\theta)+\sin (\theta)$.

## MTH 111, Review Math for Architects

Ayman Badawi

QUESTION 1. Let $u=-2 i+3 j-4 k, v=i+k, w=2 i-j+5 k$.
a) How many planes are there where each contains $u$ and $v$ and the point (1,3,2)? Find them all.
b) Is there a plane containing $u, v, w$ ? if not then find the volume of the twisted cube formed by $u, v, w$.
c) Find $\operatorname{Proj}_{w}^{u}$ and then $\left|\operatorname{Proj}_{w}^{u}\right|$.
d) Find the area of the triangle that has vertices : $(2,5),(0,8),(-3,1)$
e) Find the area of the triangle that has vertices $(1,1,2),(2,-1,1)$, and $(0,0,4)$

## Faculty information

Ayman Badawi, Department of Mathematics \& Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

MTH 111, Math for Architects,EXAM II REVIEW, Spring 2013

## Ayman Badawi

QUESTION 1. Given the points: $A=(2,8)$ and $B=(0,6)$. Find a point $C$ on the line $\mathrm{y}=3$ so that $|A C|+|C B|$ is minimum. You need to find the coordinates of the point $C$.

QUESTION 2. Find $y^{\prime}$ and don't simplify :
${ }_{\text {a) }} y=3 e^{(2 x+4)}+\ln \left(7 x^{2}+8 x+7\right)+10 x e^{2 x+1}$
») $y=4 x\left(7 x^{3}+2 e^{x}\right)^{3}+\sqrt{4 x+9}+\frac{7}{x^{3}}$
c) $y=\sqrt[3]{7 x+1}+\frac{e^{(3 x+1)}}{\ln (5 x+2)}$
d) $y=\frac{7 x+2}{x^{2}-12 x+3}$
е) $y=\ln \left[(3 x+2)^{3}\left(7 x^{2}+8 x-9\right)^{8}\right]$

ғ) $y=\ln \left[\frac{8 x^{2}+7 x-9}{4 x+e^{2 x}}\right]+3 x^{2}-45 x$
QUESTION 3. a) Find $\lim _{x \rightarrow 2} \frac{e^{(2 x-4)}-1}{3 x^{2}-3 x-6}$
b) Find $\operatorname{Lim}_{x \rightarrow-3} \frac{\sqrt[3]{3 x+1}+2}{7 x+21}$
c) $\operatorname{Lim}_{x \rightarrow-3} \frac{\sqrt[3]{3 x+1}+4}{7 x+23}$
${ }_{\text {d) }} \operatorname{Lim}_{x \rightarrow-3} \frac{\ln (3 x+10)}{e^{3 x+9}-2 x-7}$

QUESTION 4. Let $f(x)=4 x e^{2 x-3}+3 \sqrt{8 x-3}+\ln (6 x-8)-1$
a) Find the equation of the tangent line to the curve of $f(x)$ when $x=1.5$.
b) Find the actual value for $f(1.8)$ [you may want to use a calculator]
c) Use (a) to approximate $f(1.8)$.

QUESTION 5. a)Given $e^{2 x-10}+\ln (2 x+3 y)=-y x-14$. Find the equation of the tangent line to the curve at $(5,-3)$.
b) Approximate the y -value when $x=5.2$

QUESTION 6. a) Find all local min and local max of y where $y=-x^{2} e^{x}+3 e^{x}+1$.
b) For what values of $x$ does $y$ increase? for what values of $x$ does $y$ decrease?
c) Let $y$ as above but defined on $[-4,2]$ (i.e., $-4 \leq x \leq 2$ ). Find the absolute Max value of $y$ and the absolute $\min$ of $y$.

QUESTION 7. Find two numbers $A, B$ where $A+2 B=15$ and $A B$ is maximum.
QUESTION 8. We want to construct a rectangle with maximum area such that two vertices on the line $\mathrm{y}=12$ and the other two vertices on the curve $y=x^{2}$. What should be the length and the width of such rectangle?

QUESTION 9. Evaluate the following integrals:
a) $\int 7 x e^{x^{2}+1}+\sqrt{x}+4 x d x$
a/2) $\int \frac{2 x+1}{x^{2}+x+3} d x$
a/3) $\int \frac{x^{3}+x^{2}-6}{x^{7}} d x$
${ }^{\text {a/4) }} \int\left(e^{x}+1\right) \sqrt{e^{x}+x+4} d x$
a/5) $\int \frac{1}{e^{-x}+8}$
a/6) $\int \frac{2 x e^{x^{2}}}{\left(e^{x^{2}}+4\right)^{4}} d x$

## Faculty information

Ayman Badawi, Department of Mathematics \& Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates. E-mail: abadawi@aus.edu, www.ayman-badawi.com

## MTH 111, Math for Architects, Exam I Spring 2013

Ayman Badawi

QUESTION 1. a) Find an equation of the plane that contains the point $(1,2,0)$ and the two vectors : $2 i+j-k,-3 i+2 j+5 k$
b) Can we draw the line: $x=1+6 t, y=2+3 t, z=-3 k$ inside the plane in (a)? EXPLAIN CLEARLY.
c) Given a parametric equations of two lines $L_{1}$ and $L_{2}$, where $L_{1}: x=1+t, y=4-2 t, z=2+t$ and $L_{2}: x=$ $-2 s, y=9+s, z=5-6 s$. Is $L_{1}$ perpendicular to $L_{2}$ ? If the two lines intersect, then find the intersection point.

QUESTION 2. a) Given the plane $P: 3 x+y-z=14$ and $Q=(1,7,7)$ not on the plane $P$. Find the distance between $Q$ and $P$.
b) Given the line $L_{1}: x=3+2 t, y=4+t, z=2-3 t$ and $Q=(5,-3,7)$ not on the line $L_{1}$. Find the distance between $Q$ and $L_{1}$.
c) Given $V=i-2 j-2 k$. Find two vectors $W, F$ such that $W$ and $F$ are parallel to $V, W \neq F$ but $|W|=|F|=8.6$
d) Given $V=6 i-6 j+3 k$ and $W=2 i-j+2 k$. Find $\operatorname{Proj}_{V}^{W}$ and $\left|P r o j_{V}^{W}\right|$. If $\theta$ is the angle between $V$ and $W$ what is $\cos (\theta)$ ?

QUESTION 3. a)Find the area of the triangle that has vertices: $(1,1,1),(1,2,5),(2,2,7)$
b)Find the vertex, the directrix and the focus of $8 x=y^{2}-8 y+48$. Give a rough sketch of the parabola.
c) Find the center, the foci and the constant $k$ of the ellipse $4 x^{2}+8 x+y^{2}+2 y+1=0$. Give a rough sketch of the ellipse.

QUESTION 4. a) Find the center, the foci, the constant $k$ of the hyperbola $x^{2}-4 x-9 y^{2}-18 y-14=0$. Give a rough sketch of the hyperbola.
b) Find the equation of the hyperbola that has $(4,6),(4,-2)$ as its foci, and one of its vertices is $(4,4)$. Give a rough sketch of the hyperbola.

QUESTION 5. Given the points: $A=(2,8)$ and $B=(0,6)$. Find a point $C$ on the line $\mathrm{y}=3$ so that $|A C|+|C B|$ is minimum. You need to find the coordinates of the point $C$.

QUESTION 6. Find $y^{\prime}$ and don't simplify :
${ }_{\text {a) }} y=3 e^{(2 x+4)}+\ln \left(7 x^{2}+8 x+7\right)+10 x e^{2 x+1}$
в) $y=4 x\left(7 x^{3}+2 e^{x}\right)^{3}+\sqrt{4 x+9}+\frac{7}{x^{3}}$
c) $y=\sqrt[3]{7 x+1}+\frac{e^{(3 x+1)}}{\ln (5 x+2)}$
${ }_{\text {d) }} y=\frac{7 x+2}{x^{2}-12 x+3}$
е) $y=\ln \left[(3 x+2)^{3}\left(7 x^{2}+8 x-9\right)^{8}\right]$
„) $y=\ln \left[\frac{8 x^{2}+7 x-9}{4 x+e^{2 x}}\right]+3 x^{2}-45 x$

QUESTION 7. a) Find $\lim _{x \rightarrow 2} \frac{e^{(2 x-4)}-1}{3 x^{2}-3 x-6}$
b) Find $\operatorname{Lim}_{x \rightarrow-3} \frac{\sqrt[3]{3 x+1}+2}{7 x+21}$
c) $\operatorname{Lim}_{x \rightarrow-3} \frac{\sqrt[3]{3 x+1}+4}{7 x+23}$
d) $\operatorname{Lim}_{x \rightarrow-3} \frac{\ln (3 x+10)}{e^{3 x+9}-2 x-7}$

QUESTION 8. Let $f(x)=4 x e^{2 x-3}+3 \sqrt{8 x-3}+\ln (6 x-8)-1$
a) Find the equation of the tangent line to the curve of $f(x)$ when $x=1.5$.
b) Find the actual value for $f(1.8)$ [you may want to use a calculator]
c) Use (a) to approximate $f(1.8)$.

QUESTION 9 .a) (iven $e^{2 x-10}+\ln (2 x+3 y)=-y x-14$. Find the equation of the tangent line to the curve at $(5,-3)$.
b) Approximate the y -value when $x=5.2$

QUESTION 10. a) Find all local min and local max of $y$ where $y=-x^{2} e^{x}+3 e^{x}+1$.
b) For what values of $x$ does $y$ increase? for what values of $x$ does $y$ decrease?
c) Let $y$ as above but defined on $[-4,2]$ (i.e., $-4 \leq x \leq 2$ ). Find the absolute Max value of $y$ and the absolute min of $y$.

QUESTION 11. Find two numbers $A, B$ where $A+2 B=15$ and $A B$ is maximum.
QUESTION 12. We want to construct a rectangle with maximum area such that two vertices on the line $\mathrm{y}=12$ and the other two vertices on the curve $y=x^{2}$. What should be the length and the width of such rectangle?

QUESTION 13. Evaluate the following integrals:
a) $\int 7 x e^{x^{2}+1}+\sqrt{x}+4 x d x$
a/2) $\int \frac{2 x+1}{x^{2}+x+3} d x$
${ }^{\text {a/3) }} \int \frac{x^{3}+x^{2}-6}{x^{7}} d x$
${ }^{\text {a/4) }} \int\left(e^{x}+1\right) \sqrt{e^{x}+x+4} d x$
a/5) $\int \frac{1}{e^{-x}+8}$
a/6) $\int \frac{2 x e^{x^{2}}}{\left(e^{x^{2}}+4\right)^{4}} d x$
QUESTION 14. Find $y^{\prime}$ and don't simplify:
${ }_{\text {a) }} y=3 e^{(5 x+4)}+\ln \left(5 x^{2}+e^{x}+7\right)+\frac{10}{x}$
ь) $y=4 x\left(7 x^{3}+2 e^{x}\right)^{3}+\sqrt{2 x+7}+3 x^{2}$
c) $y=\sqrt[3]{7 x+1}$
f) $y=\ln \left[\frac{8 x^{2}+7 x-9}{\left(4 x+e^{2 x}\right)^{4}}\right]+10$

QUESTION 15. a) Find $\lim _{x \rightarrow-2} \frac{e^{(3 x+6)}-1}{3 x^{3}-12 x}$
b) Find $\operatorname{Lim}_{x \rightarrow 5} \frac{\sqrt{3 x+1}-4}{7 x-35}$
c) $\operatorname{Lim}_{x \rightarrow 0} \frac{3 x^{2}}{e^{x}-x-1}$

QUESTION 16. Let $f(x)=4 e^{2 x-6}+3 \sqrt{x-2}+\ln (3 x-8)-1$
a) Find the equation of the tangent line to the curve of $f(x)$ when $x=3$.
b) Use (a) to approximate $y$ when $x=2.7$.

Question 17 a. aliven $e^{x}+\ln (2 x+3 y-8)+y x+3 y-10=0$. Find he equation of the tangent line to the curve at $(0,3)$.
b) Approximate the $y$-value when $x=0.3$

QUESTION 18. a) Find the absolute maximum value of $y$ and the absolute minimum value of $y$ for $f(x)=-x e^{2 x^{2}}+e^{2 x^{2}}+1$ defined on $[-1,1]$.
b)For what values of $x$ does $f(x)$ increase? and for what values of $x$ does $f(x)$ decrease?
c) [if you like, it is only worth 2 points, i.e. if you do not feel like doing it, you only lose 2 points] Use only the concept of the first derivative and sketch a rough graph of $f(x)$

QUESTION 19. We want to construct a rectangle with maximum area such that two vertices on the line $y=4$ and the other two vertices on the curve $y=31-x^{2}$. What should be the length and the width of such rectangle?

QUESTION 20. Evaluate the following integrals:
a) $\int 7 e^{x+1}+\sqrt{x+1}+4 x d x$
a/2) $\int \frac{x+2}{x^{2}+4 x+3} d x$
а/3) $\int \frac{x^{5}+x^{6}-12}{x^{7}} d x$
${ }_{\text {a/4 }} \int\left(8 e^{x}+4\right)\left(4 e^{x}+2 x+4\right)^{7} d x$

QUESTION 21. a) A particle moves on the ellipse $x^{2}+y^{2} / 4=10$. The $y$ is decreasing at rate $0.5 \mathrm{~cm} / \mathrm{sec}$. Find the rate of change of $x$ at the point $(3,2)$.
b) a particle moves on the curve $x e^{y}+y^{2}-3 x y+\ln (3 x-8)+7 x=24$ Find the rate of change of $y$ at the point $(3,0)$ if the rate of change of $x$ is $1 \mathrm{~cm} / \mathrm{sec}$.

QUESTION 22. Find the length and the width of the largest rectangle that you can draw inside the ellipse $y^{2}+x^{2} / 4=25$ (two vertices on the x-axis and the other two vertices on the upper half of the ellipse)

QUESTION 23. Find the area of the region that is bounded by $f(x)=-x^{2}+5 x-6$ and the line $y=x-3$ where $0 \leq x \leq 4$.
QUESTION 24. Let $x^{2}+2 y^{2}+x y-4 x+8 y+4=0$. Find all the points on the curve where the tangent line at these points have slope equals 2 .

## Faculty information

Ayman Badawi, Department of Mathematics \& Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates. E-mail: abadawi@aus.edu, www.ayman-badawi.com

## MTH 111, Math for Architects, Final Exam, Summer 2013

Ayman Badawi

QUESTION 1. i) Let $w=-4 i+4 j+2 k$. Find a vector $v$ such that $\|v\|=21$ and $v \| w$.
ii) Given the line $L: x=2 t+1, y=-t+3, z=t$. Does $L$ intersect the plane $P: x+3 y-5 z=28$ in exactly one point? if yes, then find the intersection-point.
iii) Evaluate the integral $\int\left(x^{2}+x\right) e^{(x+3)} d x$

QUESTION 2. i) Evaluate the integral $\int\left(x^{3}+2 x\right) \ln (x) d x$
ii) Evaluate the integral $\int(2 x+1)\left(2 x^{2}+2 x+3\right)^{11} d x$
iii) Evaluate the integral $\int \frac{x^{2}+2 x+1}{x^{3}+3 x^{2}+3 x+2} d x$

QUESTION 3. Find the area of the region in the second quadrant of the xy-plane that is bounded by $f(x)=(x+1) e^{\left(x^{2}+2 x\right)}$, y-axis, and $-1 \leq x \leq 0$. See the figure below.


QUESTION 4. Let $f(x)=x^{2} e^{x}-3 e^{x}$
i) For what values of $x$ does $f(x)$ increase?
ii) For what values of $x$ does $f(x)$ decrease?
iii) Find all local minimum (maximum) values of $f(x)$.

QUESTION 5. Given two points $A=(10,5), B=(4,3)$. Find a point on the horizontal line $y=2$, say $C$, such that $\|A C\|+\|C B\|$ is minimum.

QUESTION 6. Given $e^{(y-1)}+e^{(x+1)}-x+4 y-7=0$. Find an equation of the tangent line to the curve at the point $(-1,1)$.

QUESTION 7. Consider the region bounded by $f(x)=(x-3)^{4}, x-a x i s, 3 \leq x \leq 4$, see the figure below, find the volume of the solid object resulted from rotating the region about the x -axis 360 degrees.


QUESTION 8. Find the absolute maximum value and the absolute minimum value of $f(x)=\left(x^{2}+2 x-3\right)^{6}$, where $-3 \leq x \leq 2$.

QUESTION 9. Given $8 x=y^{2}-6 y+25$. Find the vertex, the focus, and the directrix, then sketch

QUESTION 10. Find an equation of the ellipse where $(3,2)$ is one of the foci, $(-1,2)$ is the center, and the point $(-1,5)$ lies on the curve of the ellipse. Then find the ellipse-constant, and sketch.

QUESTION 11. Given $P_{1}:-x+4 y+2 z=10, P_{2}: 8 x+y+2 z=12$ are two planes.
a) Show that $P_{1}$ is perpendicular to $P_{2}$.
b) $P_{1}$ intersects $P_{2}$ in a line $L$. Find a parametric equations of $L$.

QUESTION 12. Find two positive numbers $x, y$ such that $x+6 y=30$ and $x y$ is maximum.

QUESTION 13. Find $y^{\prime}$ and do not simplify
i) $y=\ln \left[\frac{(3 x+2)^{5}}{(7 x+5)^{8}}\right]$
ii) $y=3 x e^{\left(x^{2}+x\right)}+\ln (6 x+1)+\frac{8}{x}$

QUESTION 14. Find $\lim _{x \rightarrow-3} \frac{e^{(x+3)}+2 x+5}{x^{3}-7 x+6}$

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Ayman Badawi, Department of Mathematics \& Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

